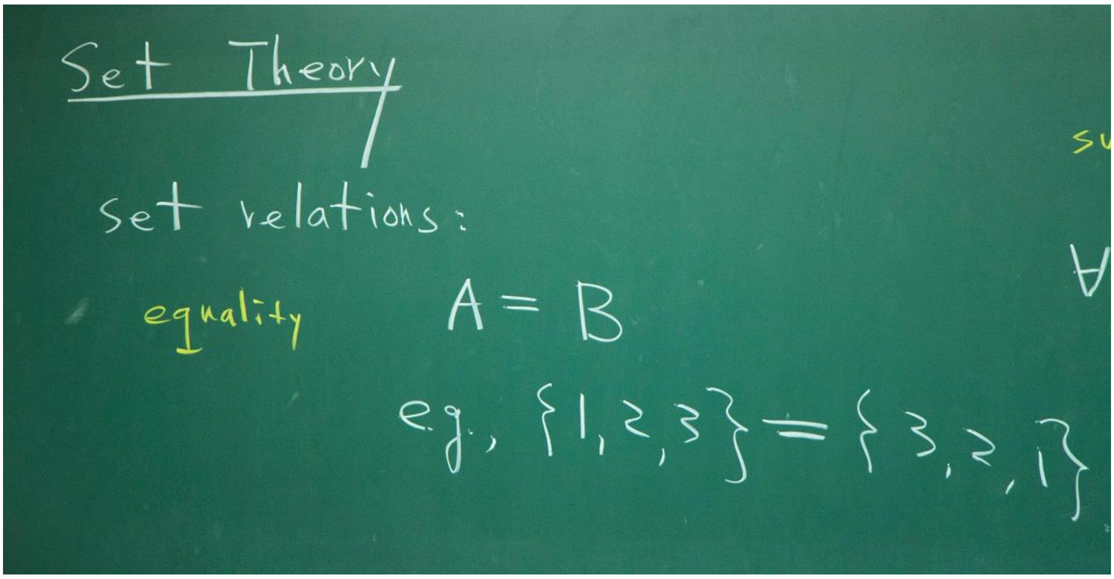
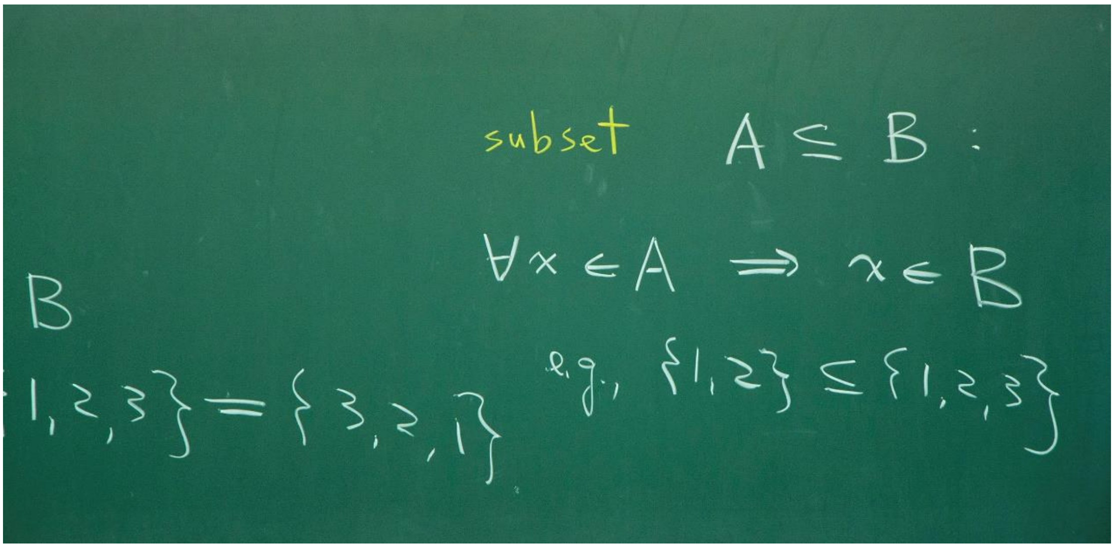
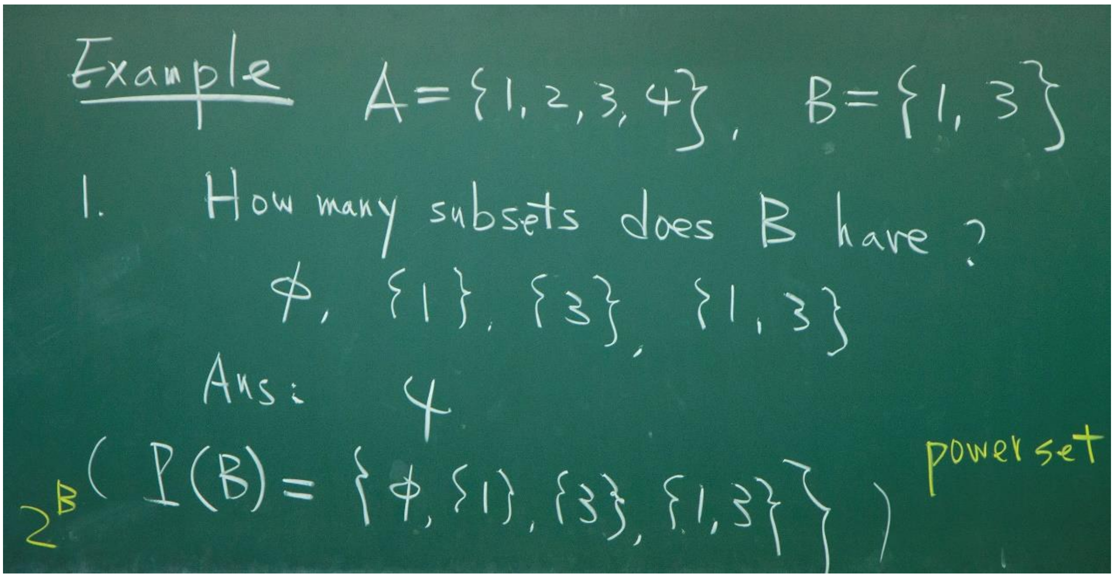
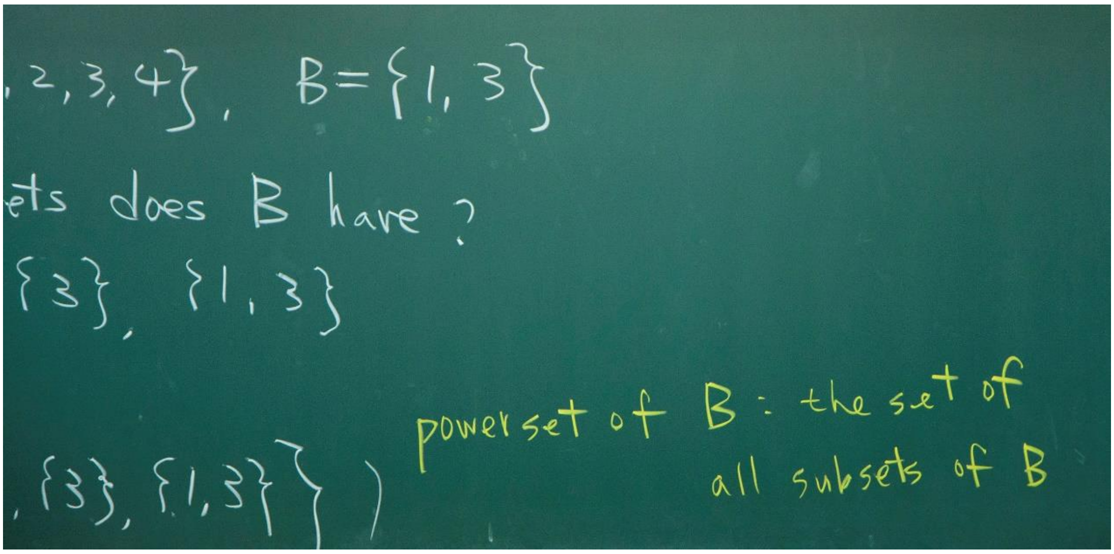
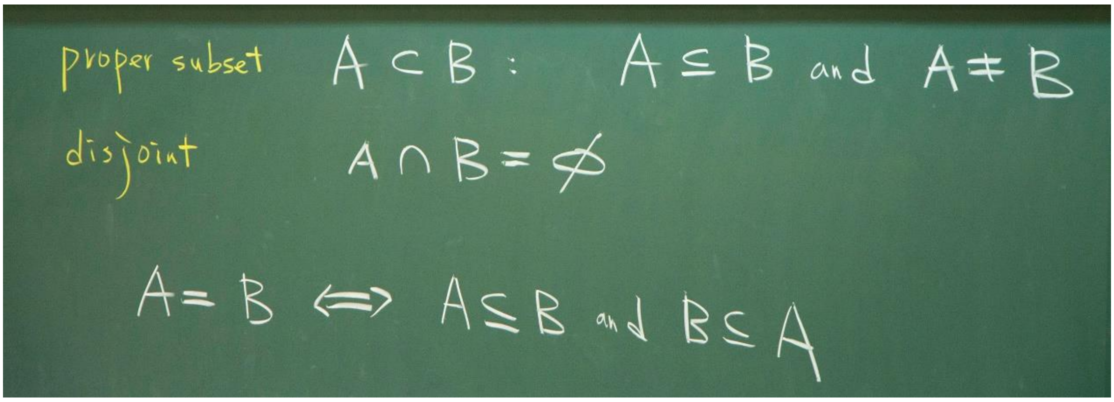
【10920 趙啟超教授離散數學 / 第 2 堂版書】

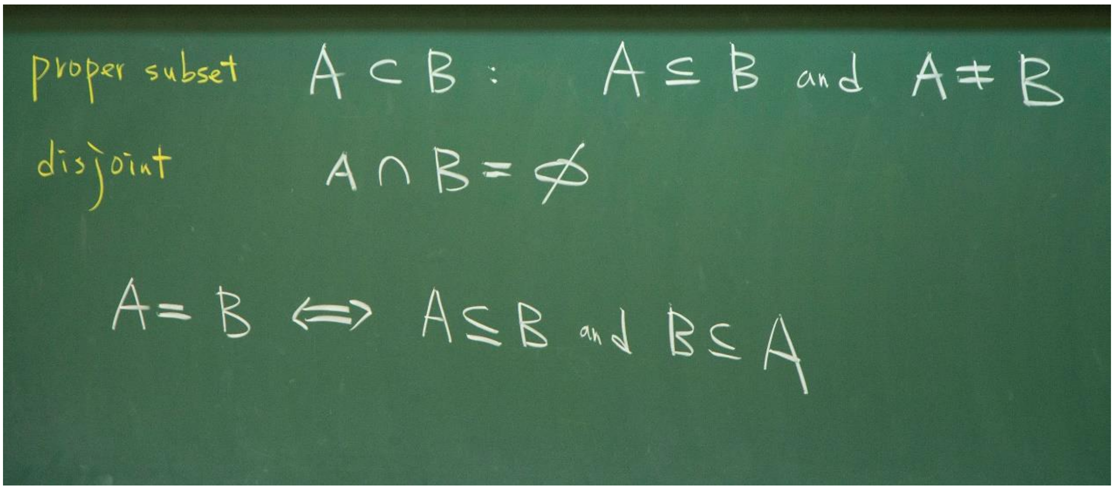







,2,3,4}, B={1,3}
ets does B have ?
{3}, {1,3}
,{3},{1,3}})
powerset of B : the set of
all subsets of B


proper subset A ⊂ B : A ⊆ B and A ≠ B
disjoint A ∩ B = ∅
A = B ⟺ A ⊆ B and B ⊆ A


proper subset A ⊂ B : A ⊆ B and A ≠ B
disjoint A ∩ B = ∅
A = B ⟺ A ⊆ B and B ⊆ A

Set Theory

set relations:

equality $A = B$

e.g., $\{1,2,3\} = \{3,2,1\}$

subset $A \subseteq B$:

$\forall x \in A \Rightarrow x \in B$

e.g., $\{1,2\} \subseteq \{1,2,3\}$

proper subset $A \subset B$: $A \subseteq B$ and $A \neq B$

disjoint $A \cap B = \emptyset$

$A = B \iff A \subseteq B$ and $B \subseteq A$

2. How many proper subsets does B have?

$\emptyset$, $\{1\}$, $\{3\}$

Ans: 3

3. How many subsets of A are disjoint from B?

$A - B = \{2, 4\}$

$\emptyset$, $\{2\}$, $\{4\}$, $\{2,4\}$ Ans: 4

Example $A = \{1, 2, 3, 4\}$, $B = \{1, 3\}$

1. How many subsets does B have?

   ϕ, $\{1\}$, $\{3\}$, $\{1, 3\}$

   Ans: 4

   2^B ($P(B) = \{\phi, \{1\}, \{3\}, \{1,3\}\}$)   power set of B: the set of all subsets of B

2. How many proper subsets does B have?

   ϕ, $\{1\}$, $\{3\}$

   Ans: 3

3. How many subsets of A are disjoint from B?

   $A - B = \{2, 4\}$

   ϕ, $\{2\}$, $\{4\}$, $\{2, 4\}$ Ans: 4

Laws that $\cap$, $\cup$, $-$ satisfy

(i) $(\bar{\bar{A}}) = A$, $A \cap \bar{A} = \phi$, $A \cup \bar{A} = \mathcal{U}$

(ii) commutative law

$A \cap B = B \cap A$, $A \cup B = B \cup A$

(iii) Associative law

$(A \cap B) \cap C = A \cap (B \cap C)$, $(A \cup B) \cup C = A \cup (B \cup C)$

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(iv) Distributive law

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

(v) DeMorgan's law

$\overline{A \cap B} = \bar{A} \cup \bar{B}$, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

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$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

(v) DeMorgan's law

$\overline{A \cap B} = \bar{A} \cup \bar{B}$, $\overline{A \cup B} = \bar{A} \cap \bar{B}$

Proof for $\overline{A \cup B} = \bar{A} \cap \bar{B}$

a. $\forall x \in \overline{A \cup B}$

$\Rightarrow x \notin A \cup B$

$\Rightarrow \neg(x \in A \cup B)$

$\Rightarrow \neg(x \in A \text{ or } x \in B)$

$\Rightarrow x \notin A \text{ and } x \notin B$

$\Rightarrow x \in \bar{A} \text{ and } x \in \bar{B} \Rightarrow x \in \bar{A} \cap \bar{B}$

$\therefore \quad \overline{A\cup B} \subseteq \overline{A} \cap \overline{B}$

b. $\quad \forall x \in \overline{A} \cap \overline{B}$

$\Rightarrow \quad x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow \quad x \notin A$ and $x \notin B$

$\Rightarrow \neg (x \in A$ or $x \in B)$

$\Rightarrow \neg (x \in A \cup B)$

$\Rightarrow \quad x \notin A \cup B$

$\Rightarrow \quad x \in \overline{A \cup B} \quad \therefore \quad \overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

Therefore, $\overline{A\cup B} = \overline{A} \cap \overline{B}$

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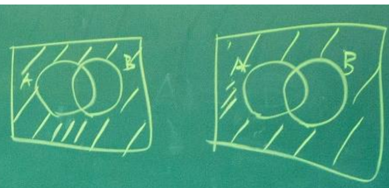

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$\Rightarrow x \notin A \cup B$

$\Rightarrow \neg(x \in A \cup B)$

$\Rightarrow \neg(x \in A$ or $x \in B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$ $\Rightarrow x \in \overline{A} \cap \overline{B}$ $\therefore \overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$

b. $\forall x \in \overline{A} \cap \overline{B}$

$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$

$\Rightarrow x \notin A$ and $x \notin B$

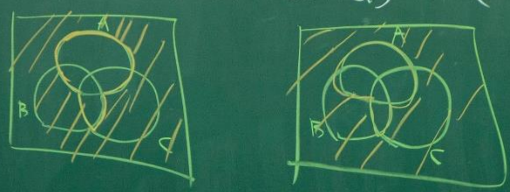
$\Rightarrow \neg(x \in A$ or $x \in B)$

$\Rightarrow \neg(x \in A \cup B)$

$\Rightarrow x \notin A \cup B$ $\Rightarrow x \in \overline{A \cup B}$ $\therefore \overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

Therefore, $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Example Show that $\overline{A \cup (B \cap C)} = (\overline{C} \cap \overline{A}) \cup (\overline{A} \cap \overline{B})$



Proof

$\overline{A \cup (B \cap C)} = \overline{A} \cap \overline{(B \cap C)}$

$= \overline{A} \cap (\overline{B} \cup \overline{C})$ (De Morgan's law)

$= (\overline{B} \cup \overline{C}) \cap \overline{A}$ (Commutative law)

$= (\overline{B} \cap \overline{A}) \cup (\overline{C} \cap \overline{A})$ (Distributive law)

$= (\overline{C} \cap \overline{A}) \cup (\overline{B} \cap \overline{A})$ (commutative law)

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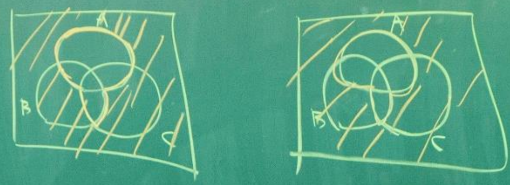

Proof

$$\overline{A \cup (B \cap C)} = \overline{A} \cap \overline{(B \cap C)} \quad \text{(De Morgan's law)}$$

$$= \overline{A} \cap (\overline{B} \cup \overline{C}) \quad \text{(De Morgan's law)}$$

$$= (\overline{B} \cup \overline{C}) \cap \overline{A} \quad \text{(Commutative law)}$$

$$= (\overline{B} \cap \overline{A}) \cup (\overline{C} \cap \overline{A}) \quad \text{(Distributive law)}$$

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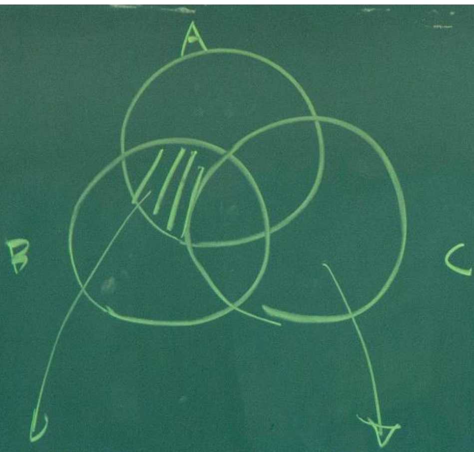
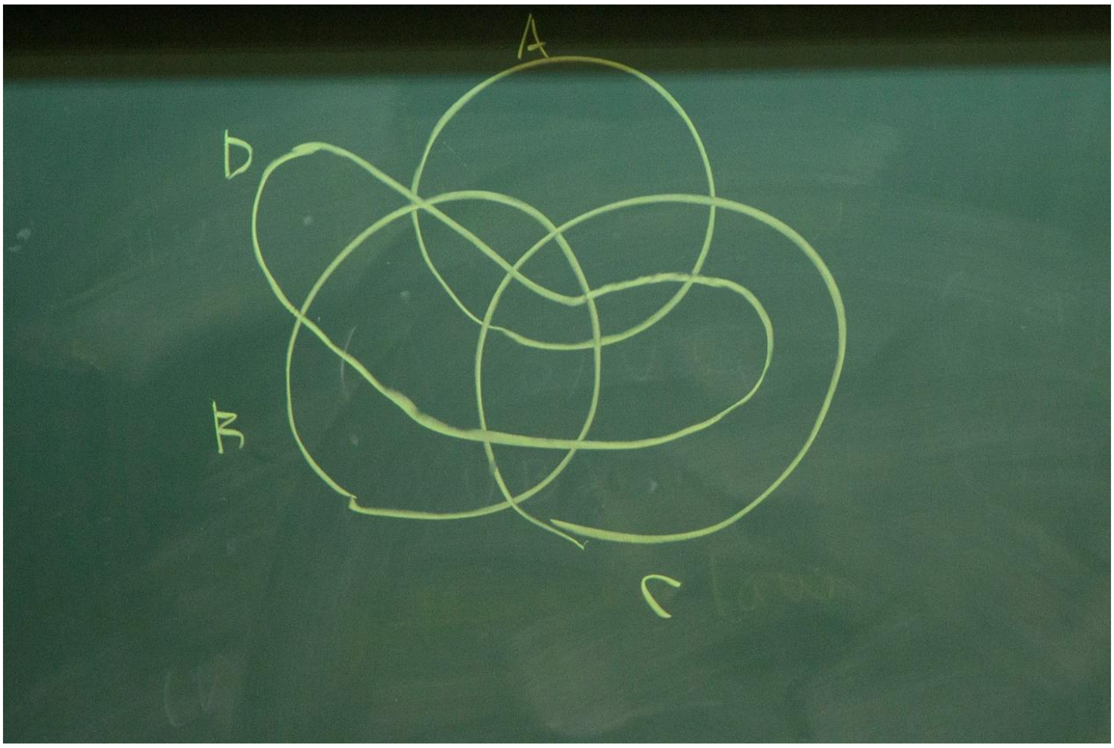
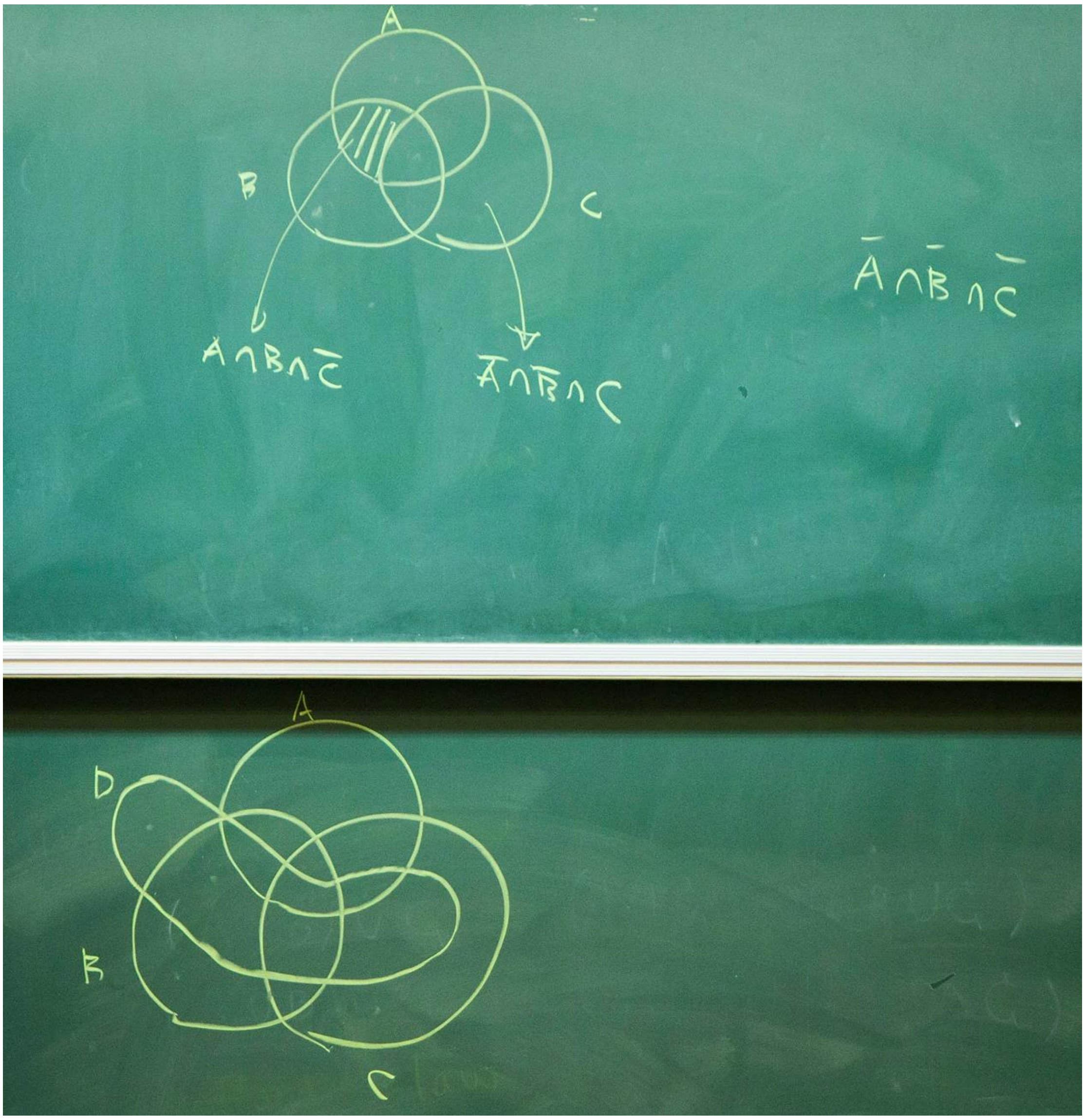


$A \cap B \cap \overline{C}$ $\quad$ $\overline{A} \cap \overline{B} \cap C$ $\quad$ $\overline{A} \cap \overline{B} \cap \overline{C}$



$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$$

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$$


A
B
C
A∩B∩C̄
Ā∩B̄∩C
Ā∩B̄∩C̄
A
D
B
C

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$$

finite set: a set consisting of a finite number of elements

infinite set: a set consisting of an infinite number of elements

If A is a finite set, then

cardinality
size $|A| \triangleq$ the number of elements in A

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$$

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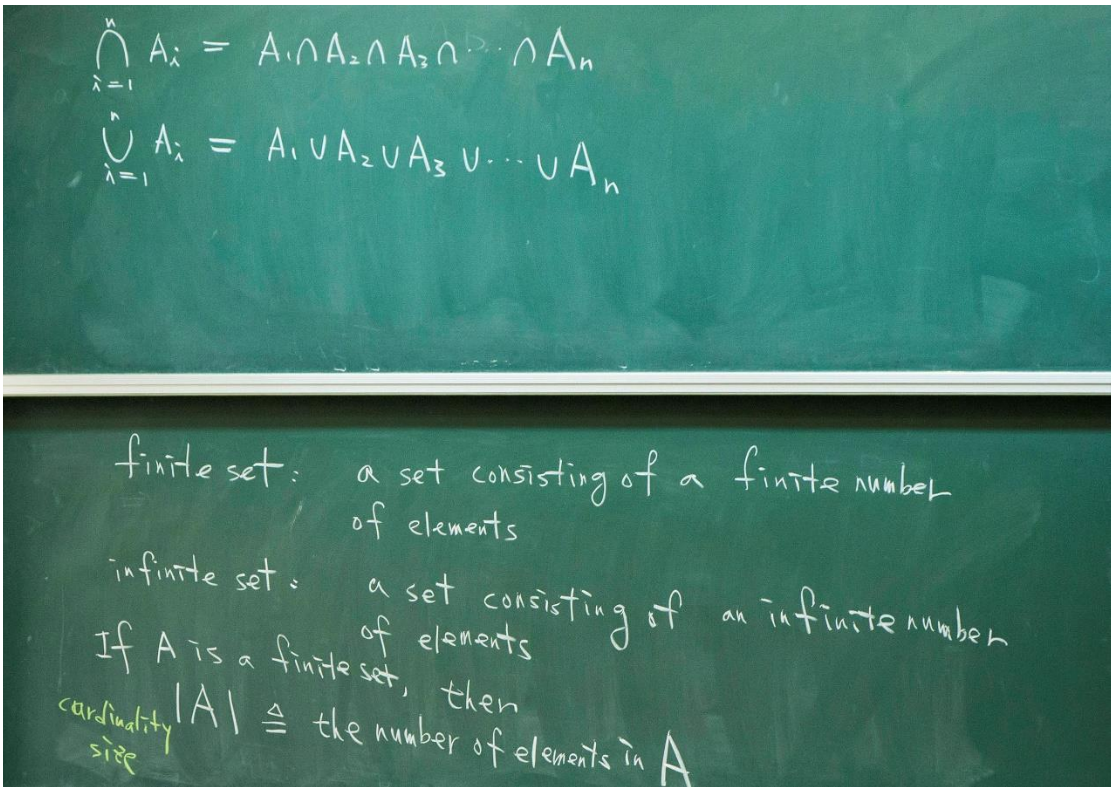
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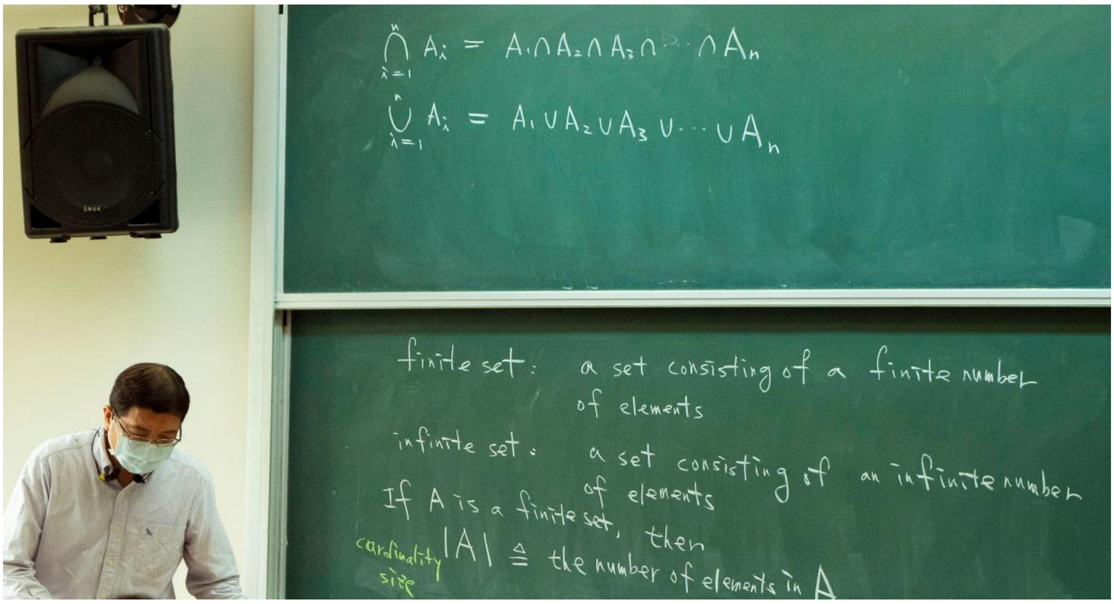
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